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Multi-objective allocation of multi-function workers with lower bounded capacity°

Albert Corominas[a], Jordi Ojeda[b], Rafael Pastor[a,*]

[a] *Research Institute IOC, Universitat Politècnica de Catalunya, Avda. Diagonal 647 pl. 11, 08028 Barcelona, Spain*

[b] *Centre CIM, Universitat Politècnica de Catalunya, c/ Llorens i Artigas 12, 08028 Barcelona, Spain*

Abstract

This paper deals with the assignment of a type of task to each member of a multi-function staff (each worker is able to perform a given subset of types of tasks, possibly with a priority index associated to each element of the subset). The resulting number of workers for each type of task must be not less than a given lower bound and as close as possible to another given value. The objectives are to minimise a function of the slacks and the surpluses of capacity, to distribute the slacks and the surpluses homogeneously among the types of task and to maximise the sum of priority indexes of the assignments. The problem is modelled as a nonlinear mixed integer program and is transformed and solved as a minimum cost flow problem.

Keywords: Manpower planning, networks

Introduction

The organization of the working time (e.g. Corominas and Crespán[1]) is nowadays a fundamental instrument to increase productivity (e.g. Cox[2]). Working time flexibility (Oke[3]) helps to adapt production capacity more closely to demand, particularly when a forecast of the

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* Corresponding author: Rafael Pastor, Research Institute IOC, Av. Diagonal 647 (edif. ETSEIB), p.11, 08028 Barcelona, Spain; e-mail addresses: rafael.pastor@upc.es

required labour for each task in every time period throughout the scheduling horizon is available.

Several authors (see, e.g. Abernathy *et al*[4] and Siferd and Benton[5]) present a hierarchical scheme for the manpower planning problems in three phases: 1) planning; 2) scheduling; 3) allocation.

For multi-function workers, the assignment of tasks is made in phase 3 (allocation), once one schedule has been assigned to each worker. But it is also possible to incorporate an allocation procedure (characteristic of phase 3) inside a heuristic or a local optimization procedure to solve the scheduling phase in each time period (e.g. one hour) into which the scheduling horizon (e.g. one week) can be divided; therefore, the allocation procedure should be very fast.

In spite of being habitual to assume that the workers can perform only one type of task (Buffa, Cosgrove and Luce[6]), there are many exceptions to this.

In Campbell and Diaby[7] a multi-department, labour-intensive service environment for allocating cross-trained workers, such as that faced by hospital nurses, is presented. The workers may have different levels of qualification in the different departments and the authors consider, as a particular case of this, the case in which the worker efficiencies are 100% of the fully qualified worker.

In Corominas, Lusa and Pastor[8] a problem of planning the staff's working hours with an annual horizon with the following characteristics is presented and solved: i) the workers are multi-functional –each category of workers is able to perform a specific subset of types of tasks; ii) different priorities exist for the allocation of a worker from a category to one type of task; and iii) all tasks are performed with the same efficiency by all workers who are capable of executing them. The multi-function is specified by means of a binary matrix (Categories/Types-of-tasks), where the element (i, j) is set to one (zero, respectively) if the workers of category i are able (or not, respectively) to perform task of type j; for each element of this matrix equal to one, a priority is specified by means of a number lying between 0 and 100.

Traditionally, in the allocation phase it is required that the total working capacity assigned to each type of task (which results from the number of workers and their efficiencies) has to be greater than or equal to given values, which depend on the desired service level. But it is worth distinguishing between the minimum service level and the goal service level, and this distinction leads us to consider, as lower bound constraints, minimum capacities and, as goals, desired capacities.

It is assumed here that, for a given period, the set of workers who are present and the minimum and desired capacities are known. Each worker must be assigned to a task in order to optimise the objectives that follow:

1) to minimise the relative shortages of actual capacities versus the desired ones, for each type of task and, moreover, to distribute them homogeneously;
2) to minimise the relative surpluses of actual capacities over desired ones and to distribute them homogeneously among the different types of tasks;
3) to maximise the priority of the allocation.

In this paper we deal with the case of worker efficiencies equal to 100% of the fully qualified worker, and with the following additional characteristics:

1) minimum capacities intervene as constraints;
2) priorities in the allocation are considered;
3) the objective function results from weighting the three aforementioned objectives.

The rest of the article is organised as follows. In section 2 the allocation problem is modelled as a nonlinear mixed integer program. In section 3 it is shown that the mathematical program can be solved as a minimum cost flow problem on a network with lower an upper bounds of the arc capacities. Section 4 includes an example and the final section contains the results of a computational experiment and the conclusions obtained.

The model

Although in some applications the problem has to be solved for a certain number of consecutive periods, here we consider a unique period (e.g. an hour or a week) and therefore, the period index is omitted.

Data:

C	Number of categories of workers.
N_c	Number of workers of category *c (c=1...C)*.
W	$=\sum_{c=1}^{C} N_c$, number of members of the staff.
K	Number of types of tasks (or functions).
F	Binary matrix of multi-function: f_{ck} = 1 if the workers of category *c (c=1...C)* can perform tasks of type *k (k=1...K)*.
P	Matrix of priorities: p_{ck} is the priority that a worker of the category *c (c=1...C)* performs a task of type *k (k=1...K)*, $\forall(c,k) \mid f_{ck}=1$
DM_k	Integer lower bound of the number of workers (capacity) assigned to task type *k (k=1...K)*.
D_k	Positive integer desired value of the number of workers (capacity) assigned to task type *k (k=1...K)*.
β, λ	Parameters to weigh the different parts of the objective function $(\beta+\lambda\le 1)$.
γ_k	Relative importance of the shortage of task type *k (k=1...K)*.
μ_k	Relative importance of the surplus of task type *k (k=1...K)*.

Decision variables:

x_{ck} Number of workers of category *c (c=1...C)* allocated to task type *k (k=1...K),* $\forall(c,k) \mid f_{ck} = 1$

Derived variables:

δ_k^- Shortage for task type *k (k=1...K)*:

$$\delta_k^- = \max \left\{0,\ D_k - \sum_{\forall c \mid f_{ck}=1} x_{ck}\right\} \qquad k = 1...K$$

δ_k^+ Surplus for task type *k (k=1...K)*:

$$\delta_k^+ = \max \left\{0,\ \sum_{\forall c \mid f_{ck}=1} x_{ck} - D_k\right\} \qquad k = 1...K$$

The problem can now be stated as follows:

minimise

$$Z = \beta \cdot \left[\sum_{k=1}^{K} \gamma_k \cdot \Phi\,(D_k, DM_k, \delta_k^-)\right] + \lambda \cdot \left[\sum_{k=1}^{K} \mu_k \cdot \Omega\,(D_k, \delta_k^+)\right] - (1 - \beta - \lambda) \cdot \left[\sum_{\forall(c,k) \mid f_{ck}=1} p_{ck} \cdot x_{ck}\right] \quad (1)$$

subject to

$$\sum_{\forall c \mid f_{ck}=1} x_{ck} + \delta_k^- - \delta_k^+ = D_k \qquad \mathrm{k = 1...K} \qquad (2)$$

$$\sum_{\forall k \mid f_{ck}=1} x_{ck} = N_c \qquad \mathrm{c = 1...C} \qquad (3)$$

$$x_{ck} \geq 0 \text{ and integer} \qquad \forall(c,k) \mid f_{ck} = 1 \qquad (4)$$

$$\delta_k^-, \delta_k^+ \geq 0 \qquad k = 1...K \qquad (5)$$

Equation (1) is the objective function which includes the penalties associated with the shortages and the surpluses and the bonus (with negative sign) associated with the priority of the allocation of employees of the staff to the different types of tasks; (2) imposes, for each type of task, that the total number of the workers assigned to a type of task be equal to the required labour plus, if it is the case, the shortage or minus, if it is the case, the surplus; (3) is the balance between the available presence from a specific type of staff workers and those assigned to the different types of tasks; (4) expresses the non-negative integer character of the corresponding variables; and (5) expresses the non-negative character of the corresponding variables (in the optimal solution these variables will take integer values).

The use of a nonlinear convex function $\Phi\,(D_k\,, DM_k\,, \delta_k^-)$ in the objective function is multipurpose. This function allows us to minimise the relative shortages and to distribute them homogeneously among the different types of tasks (it is assumed that a solution in which the shortages are regularly distributed among the tasks is preferred to another in which the total shortage accumulates exclusively in one or in some few types of tasks); i.e. to approach the goal service level. Moreover, function Φ avoids, when it is possible, infeasible solutions in which the lower bound constraints on the capacities are not fulfilled; i.e. to try to guarantee the minimum service level. The function is defined as follows:

Let:

$$\Psi_k = \frac{\delta_k^-}{D_k} \tag{6}$$

and

$$\varphi(D_k\,, \delta_k^-) = \left(\frac{\Psi_k}{1 - \Psi_k + \varepsilon} \right) \cdot D_k \tag{7}$$

where ε is a small positive number, then

$$\Phi(D_k, DM_k, \delta_k^-) = \begin{cases} \varphi\left(D_k, \delta_k^-\right) & if\ (\mathrm{D_k} - \delta_k^-) \geq \mathrm{DM_k} \\ \varphi\left(D_k, D_k - DM_k\right) + M \cdot \left[\varphi\left(D_k, \delta_k^-\right) - \varphi\left(D_k, D_k - DM_k\right)\right] & if\ (\mathrm{D_k} - \delta_k^-) < \mathrm{DM_k} \end{cases} \quad (8)$$

Function Φ is equal to function φ except when the workers assigned to a type of task $(D_k - \delta_k^-)$ do not achieve the minimum capacity (DM_k); in that case the slopes of function φ are multiplied by M. The value of the penalty coefficient, M, must be great enough to make any solution fulfilling the lower bound constraints preferable to any other which violate them.

In a similar way, a nonlinear convex function, $\Omega(D_k, \delta_k^+)$, has been used in order to minimise the relative surpluses and to distribute them homogeneously among the different types of tasks:

Let:

$$\Gamma_k = \frac{\delta_k^+}{D_k + \delta_k^+} \quad (9)$$

then

$$\Omega(D_k, \delta_k^+) = \left(\frac{\Gamma_k}{1 - \Gamma_k + \varepsilon'}\right) \cdot \left(D_k + \delta_k^+\right) \quad (10)$$

where ε’ is a small positive number.

The values of ε and ε' avoid divisions by 0 and must be small enough to obtain sufficiently large values when $\Psi_k = 1$ or $\Gamma_\mathrm{k} \to 1$.

The objective function has been presented as continuous but, owing to the fact that variables are restricted to be integer (the variables δ_k^- and δ_k^+ are not integer, but they will take integer values at optimality), the objective function can be considered as a piecewise-linear one; therefore an integer mathematical program with piecewise-linear convex cost function is obtained.

The solution of the mathematical program would provide the number of workers of each category allocated to each type of task and the shortage or the surplus corresponding to each type of task.

Solving the model as a minimum cost network flow

The described model is a nonlinear mixed integer mathematical program whose difficulty to be solved is well-known; but, as has been commented, a convex piecewise- linear model can be obtained. General transformations of such problems to convex cost network flow problems are well-known (see, e.g. Ahuja, Magnanti and Orlin[9]); therefore we propose to transform and solve it as a minimum cost flow problem in a particular network with lower and upper bounded arc capacities with appropriate unit transport costs.

The network includes the nodes that follow:

Node C_c *(c=1...C)* for each category of workers.

Node T_k *(k=1...K)* for each type of task.

Node $I_{d_k}T_k$ (with $d_k = 1...D_k$) for each unit of desired capacity for each type of task *(k=1...K)*.

Node $U_{s_k}T_k$ (with $s_k = 1...\max\left(0, \sum_{\forall c \mid f_{ck}=1} N_c - D_k\right)$) for each unit of possible surplus for each type of task *(k=1...K)*.

The node set is completed by defining a source, α, and a sink, ω.

The arcs, their lower and upper bounds of the capacities and their unitary costs, are defined in table 1:

[Insert table 1 about here]

To assign one unit of flow to the arc ($I_{d_k}T_k$,T_k) corresponds to one *unit of shortage* associated with the origin node ($I_{d_k}T_k$) in order to feed the flow of D_k units which goes from node T_k to sink ω.

In a similar way, to assign one unit of flow to the arc (T_k,$U_{s_k}T_k$) corresponds to one *unit of surplus* associated with the destination node ($U_{s_k}T_k$).

Example

Next, an example of the designed model is shown:

W = 5; C = 2; K = 3; N = [3, 2]; F = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$; P = $\begin{bmatrix} 100 & 25 & - \\ - & 50 & 100 \end{bmatrix}$; DM = [0, 1, 2]; D = [2, 3, 2]; β = 0.9; λ = 0.09; $\gamma = \mu$ = [1, 1, 1]; M = 10 000; $\varepsilon = \varepsilon'$ = 0.001

Figure 1 presents the network representation and table 2 shows some of the arcs with their minimum/maximum capacities and their unitary cost. The values of ε and ε' were found to be suitable for the pursued objective.

[Insert figure 1 about here]

[Insert table 2 about here]

The obtained results are:

$$X = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix};\ \delta^{-} = [1, 1, 0];\ \delta^{+} = [0, 0, 0]$$

If instead of using β = 0.9 and λ = 0.09 we used β = 0.49 and λ = 0.49, the obtained results would be the following:

$$X = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix};\ \delta^{-} = [0, 2, 0];\ \delta^{+} = [0, 0, 0]$$

For greater values of the number of members of the staff W = 10 and N = [6, 4], the following solutions would be obtained:

$$\beta = 0.9 \text{ and } \lambda = 0.09\text{: } X = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix};\ \delta^{-} = [0, 0, 0];\ \delta^{+} = [3, 0, 0]$$

$$\beta = 0.49 \text{ and } \lambda = 0.49\text{: } X = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix};\ \delta^{-} = [0, 0, 0];\ \delta^{+} = [2, 0, 1]$$

We can see the effect of the different components of the objective function, changing the weight associated to each one of them. If any objective is hierarchical regarding the others, it is easy to calculate the value of parameters β and λ in order to impose the hierarchical structure.

Results and conclusions

To generate and to solve the minimum cost flow problem, an application in Microsoft Visual C 6.0 has been developed and the ILOG-CPLEX Network Optimizer 7.5 library has been used. The computations have been performed with a PC Pentium III at 1100 Mhz with 512 Mb of RAM.

A full computational experiment has been performed, with the following testing set:

W = 25, 50, 100 and 250 workers.

(C , K) = (2 , 3), (3 , 3) and (4 categories , 5 tasks).

For each one of the 12 possible combinations of W and (C , K), 10 test-problems have been obtained varying the required labour of the tasks, D_k.

In all the test-problems, the calculation time is less than 0.001 seconds (see the average times in table 3). Specifically: for W = 100 and (C , K) = (3 , 3), the average calculation time is 0.00014 seconds.

[Insert table 3 about here]

The calculation time depends on the size of the graph: the number of nodes and arcs can be obtained through the following expressions:

$$Nodes = 2 + C + K + \sum_{k=1}^{K} D_k + \sum_{k=1}^{K} \left(\max \left\{ 0, \sum_{\forall c |\, f_{ck}=1} N_c - D_k \right\} \right) \qquad (11)$$

$$Arcs = C + K + 2 \cdot \sum_{k=1}^{K} D_k + \sum_{c=1}^{C} \sum_{k=1}^{K} f_{ck} + 2 \cdot \sum_{k=1}^{K} \left(\max \left\{ 0, \sum_{\forall c |\, f_{ck}=1} N_c - D_k \right\} \right) =$$

$$= 2 \cdot Nodes - 4 - \mathrm{C} - \mathrm{K} + \sum_{c=1}^{C} \sum_{k=1}^{K} f_{ck} \qquad (12)$$

If the results of the 120 test-problems are analyzed, a relationship between the number of nodes and the calculation time can be established (see figure 2). This relationship is approximately linear and it can be expressed with the following expression (the correlation coefficient is 0.99743):

$$Time(ms) = 0.00059693 \cdot Nodes - 0.00811065 \qquad (13)$$

[Insert figure 2 about here]

Since an almost linear relationship exists between the number of nodes and that of arcs, see expression (12), obviously the relationship between the number of arcs and the calculation time is also approximately linear and it can be expressed with the following expression (the correlation coefficient is 0.99758):

$$Time(ms) = 0.00029799 \cdot Arcs - 0.00723034 \qquad (14)$$

In applications it will be usual to have fewer than 100 workers and a weekly scheduling horizon of, for example, 72 periods (1 period/hour, 12 hours/day and 6 days/week). The results allow the authors to emphasize the feasibility of the proposed allocation procedure to be used in the resolution of problems of real dimensions, even inside a heuristic or a local optimization procedure to assign schedules.

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Arc	**Minimum capacity**	**Maximum capacity**	**Unitary cost**
(α,C_c)	N_c	N_c	0
$(\alpha, I_{d_k}T_k)$	0	1	0
(C_c,T_k) $\forall(c,k) \mid f_{ck}=1$	0	N_c	$-(1-\beta-\lambda)\cdot p_{ck}$
$(I_{d_k}T_k, T_k)$	0	1	$\beta\cdot\gamma_k\cdot\left[\Phi\left(D_k, DM_k, D_k-d_k+1\right)-\Phi\left(D_k, DM_k, D_k-d_k\right)\right]$
$(T_k, U_{s_k}T_k)$	0	1	$\lambda\cdot\mu_k\cdot\left[\Omega(D_k,s_k)-\Omega(D_k,s_k-1)\right]$
(T_k,ω)	D_k	D_k	0
$(U_{s_k}T_k,\omega)$	0	1	0

Table 1. The arcs of the network with their lower and upper bounds of the capacities and their unitary costs

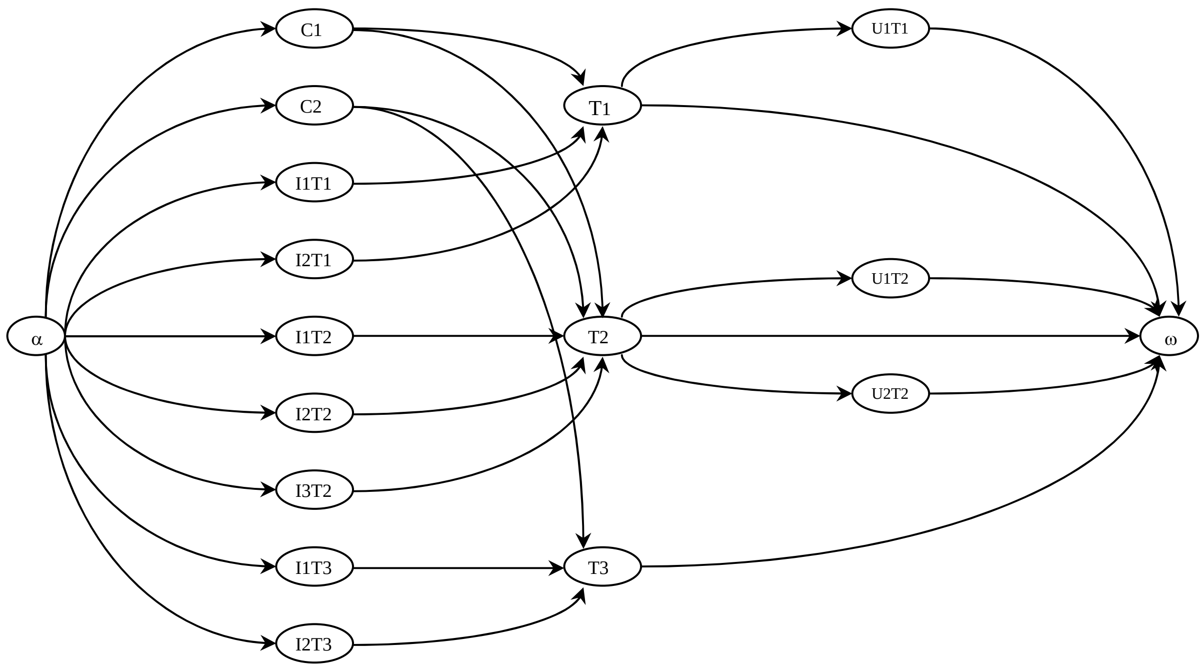

Figure 1. Network diagram of the example.

Arc	**Minimum capacity**	**Maximum capacity**	**Unitary cost**
(α, C_1)	3	3	0
(α, I_1T_3)	0	1	0
(C_2, T_2)	0	2	-(1-0.9-0.09)·50 = -0.5
(I_1T_2, T_2)	0	1	$0.9 \cdot 1 \cdot \left[\Phi(3,1,3) - \Phi(3,1,2)\right]$ = 26 946 161.52
(T_2, U_2T_2)	0	1	$0.09 \cdot 1 \cdot \left[\Omega(3,2) - \Omega(3,1)\right]$ = 0.18
(T_3, ω)	2	2	0
(U_1T_1, ω)	0	1	0

Table 2. Diverse arcs with their minimum/maximum capacities and their unitary costs

Number of workers

(C , K)	25	50	100	250
(2 , 3)	$0.03\ 10^{-3}$	$0.06\ 10^{-3}$	$0.11\ 10^{-3}$	$0.29\ 10^{-3}$
(3 , 3)	$0.03\ 10^{-3}$	$0.07\ 10^{-3}$	$0.14\ 10^{-3}$	$0.34\ 10^{-3}$
(4 , 5)	$0.06\ 10^{-3}$	$0.11\ 10^{-3}$	$0.23\ 10^{-3}$	$0.58\ 10^{-3}$

Table 3. Average calculation times [s].

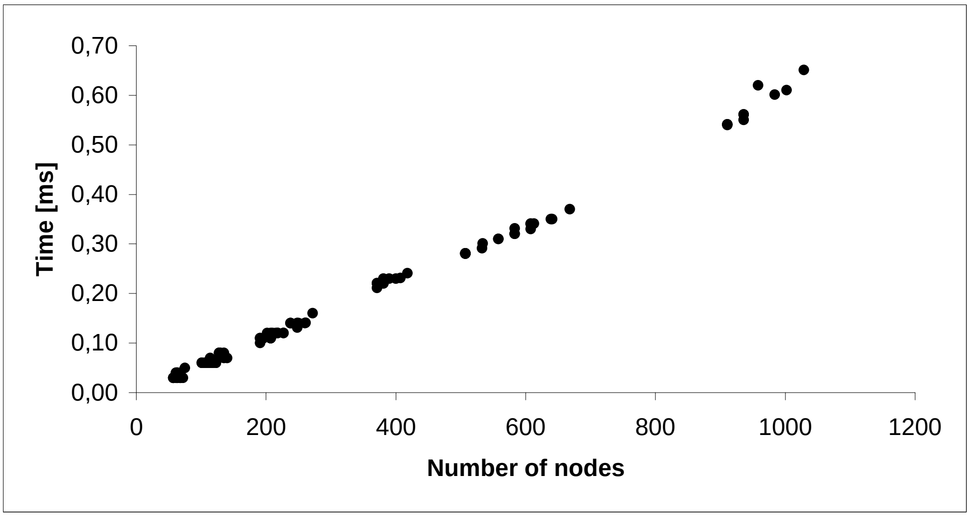

Figure 2. Calculation time versus number of nodes.

Figure captions and table headings

Figure 1. Network diagram of the example.

Figure 2. Calculation time versus number of nodes.

Table 1. The arcs of the network with their lower and upper bounds of the capacities and their unitary costs

Table 2. Diverse arcs with their minimum/maximum capacities and their unitary costs

Table 3. Average calculation times [s].